

(Time: 3 hours)

Max. Marks: 80

- N.B. (1) Question No. 1 is compulsory.
 (2) Answer any three questions from Q.2 to Q.6.
 (3) Use of Statistical Tables permitted.
 (4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $t \sqrt{1 + \sin t}$ [5]

(b) Find the constants a, b, c, d, e if [5]

$$f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y) \text{ is analytic.}$$

(c) Calculate the Spearman's rank correlation coefficient R [5]

X : 85, 74, 85, 50, 65, 78, 74, 60, 74, 90

Y : 78, 91, 78, 58, 60, 72, 80, 55, 68, 70

(d) Find inverse Laplace transform of $\tan^{-1} \left(\frac{s+a}{b} \right)$. [5]

Q2.

(a) Find the Laplace transform of $e^{-4t} \int_0^t u \sin 3u du$ [6]

(b) find the value of k if the function $f(x) = kx^2(1-x^3)$, $0 \leq x \leq 1$.

$$F(x) = 0 \quad \text{otherwise}$$

Is a probability density function. find mean and variance. [6]

(c) Obtain the Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$ [8]

Q3.

(a) Find the analytic function $f(z) = u + iv$ such that [6]

$$u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x} .$$

(b) Using convolution theorem Find inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+4)}$. [6]

(c) Fit a second-degree parabolic curve to the following data

Year (x)	: 1974	1975	1976	1977	1978	1979	1980	1981
Production (y)	: 12	14	26	42	40	50	52	53.

[8]

Q4.

(a) Obtain the Fourier series to represent $f(x) = 9 - x^2$ in $(-3, 3)$. [6]

(b) . Find the coefficients of regression and hence obtain the equation of the lines of Regression for the following data

X: 78, 36, 98, 25, 75, 82, 90, 62, 65, 39.

Y: 84, 51, 91, 60, 68, 62, 86, 58, 53, 47. [6]

(c) Prove that $\int_0^\infty e^{-t} \frac{\sin 2t + \sin 3t}{t} dt = \frac{3\pi}{4}$. [8]

Q5.

(a) Find the orthogonal trajectories of the family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$. [6]

(b) If X denotes the outcome when a fair die is tossed, find Moment generating function Of X and hence find the mean and variance of X. [6]

(c) Obtain the half range cosine series of $f(x) = x(\pi - x)$ in $(0, \pi)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$ [8]

Q6.(a) Find inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. [6]

(b) The probability density function of a random variable X is

X	: 0	1	2	3	4	5	6
P (X = x)	: k	3k	5k	7k	9k	11k	13k

Find k , $p(X < 4)$, $P(3 < X \leq 6)$. [6]

(c) Verify Laplace equation for $u = \left(r + \frac{a^2}{r}\right) \cos \theta$. also find v and $f(z)$. [8]

(3 Hours)

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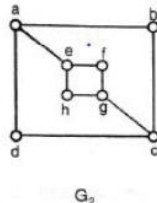
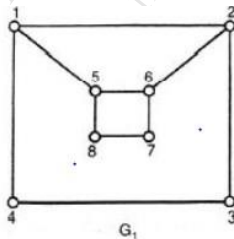
- N.B. : (1) Question Number 1 is compulsory
 (2) Solve any three questions from the remaining questions
 (3) Make suitable assumptions if needed
 (4) Assume appropriate data whenever required. State all assumptions clearly.

1. a. Define the following with suitable example 5
 a) Ring b) Cyclic Group c) Monoid d) Normal Subgroup e) Planar Graph
 b. Check whether $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology 5
 c. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2,3 or 5. 5
 d. Prove by mathematical induction that 5
 $2+5+8+\dots+(3n-1) = n(3n+1)/2$
 2 a Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on $A \times A$ defined by $(a,b) R (c,d)$ if and only if $ad = bc$. Prove that R is an Equivalence Relation 8
 b. Let $A = \{a, b, c, d, e\}$ 8

$$MR = \begin{matrix} & a & b & c & d & e \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & 1 & 0 & 1 & 1 & 0 \\ & 0 & 1 & 0 & 0 & 1 \\ & 0 & 1 & 0 & 0 & 1 \\ & 1 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 1 \end{matrix}$$

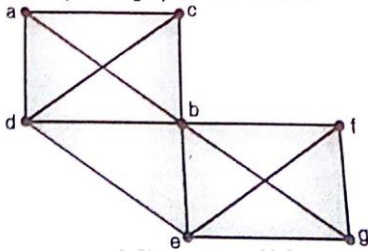
Find the transitive closure of it using Warshall's algorithm.

- c Let G be a group. Prove that the identity element 'e' is unique. 4
 3 a Prove that set $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication module 7 8
 b Give the exponential generating function for the sequences 8
 i) $\{1,1,1,\dots\}$
 ii) $\{0,1,0,-1,0,1,0,-1,\dots\}$.
 c Determine whether the following graphs are isomorphic. Justify your answer. 4



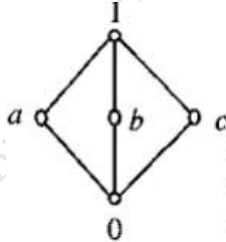
4. a A Function $f: R - \left\{ \begin{matrix} 7 \\ 3 \end{matrix} \right\} \rightarrow R - \left\{ \begin{matrix} 4 \\ 3 \end{matrix} \right\}$ is defined as $f(x) = (4x - 5)/(3x - 7)$ 8
 Prove that f is Bijective and find the rule for f^{-1}

- b Show that $(2,5)$ encoding function $e: \mathbf{B}^2 \rightarrow \mathbf{B}^5$ defined by
 $e(00)=00000$
 $e(01)=01110$
 $e(10)=10101$
 $e(11)=11011$
 is a group code. 8
- c Check whether Euler cycle and Euler Path exist in the Graph given below. If yes Mention them 4



5. a Consider the Set $A=\{1,2,3,4,5,6\}$ under multiplication Modulo 7. 8
 1) Prove that it is a Cyclic group.
 2) Find the orders and the Subgroups generated by $\{2,3\}$ and $\{3,4\}$
- b State and explain the extended Pigeonhole principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. 8
- c Functions f, g, h are defined on a set $X=\{a, b, c\}$ as 4
 $f=\{(a,b), (b,c), (c,a)\}$
 $g=\{(a,b), (b,a), (b,b)\}$
 $h=\{(a,a), (b,b), (c,a)\}$
 i) Find $f \circ g$, $g \circ f$. Are they equal?
 ii) Find $f \circ g \circ h$ and $f \circ h \circ g$?
6. a Draw the Hasse Diagram of D_{72} and D_{105} and check whether they are Lattice. 8

- b Define Bounded Lattice and Distributive Lattice. Check if the following diagram is a Distributive Lattice or not 8



- c Define the following with suitable example. 4
 a) Hamiltonian path b) Euler Circuit c) Sub Lattice d) Group e) Surjective Function

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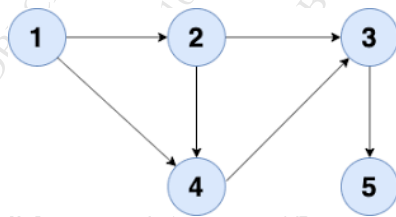
N.B: (1) Question No. 1 is compulsory

(2) Attempt any three questions out of the remaining five questions

- Q.1 (a) Explain various types of data structures with example. **5**
 (b) Define Graph and explain various graph representation techniques. **5**
 I Convert the following expression to postfix. **5**
 $(f-g) * ((a+b) * (c-d))/e$
 (d) Differentiate between B tree and B+ tree. **5**

- Q.2 (a) Apply linear probing and quadratic probing hash functions to insert values in the Hash table of size 10. Show number of collisions occurs in each technique. **10**
 27, 72, 63, 42, 36, 18, 29, 101
 (b) Construct B+ tree of order 3 for the following dataset **10**
 90, 27, 7, 9, 18, 21, 3, 4, 16, 11, 1, 72

- Q.3 (a) Write BFS algorithm. Show BFS traversal for the following graph with all the steps.



- (b) Write a C program to implement linear queue using array. **10**
- Q.4 (a) Write a program to perform the following operations on the Singly linked list: **10**
 i. Insert a node at the end
 ii. Delete a node from the beginning
 iii. Search for a given element in the list
 iv. Display the list
 (b) Write a C program to implement Stack using Linked List **10**

- Q.5 (a) Write a program to evaluate postfix expression using stack data structure **10**
 (b) Construct AVL for following elements **10**
 50, 25, 10, 5, 7, 3, 30, 20, 8, 15

- Q.6 (a) Construct Binary Tree from following traversal. **10**
 In-order Traversal: D B H E I A F J C G
 Post order Traversal: D H I E B J F G C A
 (b) Write a C program for polynomial addition using a Linked-list. **10**

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Q1	A	Convert	05
		i) 147 in to binary	
		ii) $(23A)_{16}$ in to Decimal	
		iii) $(135)_8$ in to decimal	
		iv) 234 in to BCD	
		v) 23 in to gray code	
	B	Write a short note on Encoder	05
	C	Differentiate between Hardwired control unit and Micro programmed control unit	05
	D	Differentiate between SRAM & DRAM	05
Q2	A	Draw the flow chart of Non Restoring division algorithm and Perform $4 \div 2$	10
	B	Explain Flynn's classification	10
Q3	A	Explain the instruction cycle with the help of a neat state diagram	10
	B	Explain the various addressing modes	10
Q4	A	Using booth's algorithm perform -5×-3	10
	B	Represent -786.25 using IEEE 754 standards (both single and double precision format)	10
Q5	A	Explain different memory Mapping Techniques	10
	B	List & Explain the Characteristics of Memory	05
	C	What do you mean by cache coherence	05
Q6	A	Draw and explain 4 stage instruction pipelining and briefly describe the hazards associated with it	10
	B	Describe various Bus Arbitration methods	10

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 2) Attempt any three out of the remaining.
 3) Assume suitable data if necessary and justify the assumptions.
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- Q1 a)** What are homogeneous coordinates? Write a homogenous transformation matrix for translation, scaling, and rotation. [05]
- b)** Explain the working of the Raster scan system with a neat diagram, [05]
- c)** Explain any 5 principles of animation. [05]
- d)** Scale a triangle A(4,4), B(12,4) and C(8,10) with scaling factor $S_x=2$ and $S_y=1$. [05]
- Q2a)** Write a midpoint circle drawing algorithm. Apply this algorithm to find pixel coordinates of the circular boundary only for the first quadrant, whose radius is 8 units. [10]
- b)** Rotate a line segment with endpoint A (3,3) to B(10,10) in a clockwise direction by an angle 45 degrees by keeping A (3,3) as fixed point. Find new transformed coordinates of a line. [10]
- Q3a)** Explain Flood fill and boundary fill algorithm with a suitable example. Write merits and demerits of the same. [10]
- b)** Derive transformation matrix for 2D rotation about a fixed point. [10]
- Q4 a)** Explain the z-buffer algorithm for hidden surface removal with a suitable example. [10]
- b)** Explain Sutherland-Hodgeman polygon clipping algorithm with a suitable example. [10]
- Q5 a)** What is Bezier curve? Write important properties of the Bezier curve. [10]
- b)** What do you mean by line clipping? Explain Cohen-Sutherland line clipping algorithm with a suitable example. [10]
- Q6 a)** Write a note on 3D projections. [05]
- b)** What is animation? Explain key frame animation. [05]
- c)** What are the properties of fractals? Explain how the Koch curve is constructed. Calculate the dimensions of Koch curve. [05]
- d)** What do you mean by aliasing? Explain any two Anti-aliasing techniques. [05]
